Simultaneous Equations

1.

Find the coordinates of the points of intersection of the given straight line and curve in each case.

a $y = x + 2$, $y = x^2 - 4$

b $y = 4x + 11$, $y = x^2 + 3x - 1$

c $y = 2x - 1$, $y = 2x^2 + 3x - 7$

2.

The line $y = 5 - x$ intersects the curve $y = x^2 - 3x + 2$ at the points P and Q.

Find the length PQ in the form $k\sqrt{2}$.

3.

Solve the simultaneous equations

$$3^{x-1} = 9^{2y}$$
$$8^{x-2} = 4^{1+y}$$

4.

Use an algebraic method to show that the graphs

$$y = 1 - x \quad \text{and} \quad y = x^2 - 6x + 10,$$

do **not** intersect.

5.

The line straight L and the curve C have respective equations

$$L: \ 2y = 7x + 10,$$

$$C: \ y = x(6 - x).$$

a) Show that L and C do not intersect.

b) Find the coordinates of the maximum point of C.

c) Sketch on the same diagram the graph of L and the graph of C, showing clearly the coordinates of any points where each of the graphs meet the coordinate axes.

6.

The curve C has equation

$$y = 4x^2 - 7x + 11.$$

The straight line L has equation

$$y = 5x + k,$$

where k is a constant.

Given that C and L intersect at two distinct points, show that $k > 2$.

7.

The straight line L has equation

$$y = kx - 9,$$

where k is a constant.

The curve C has equation

$$y = 3(x+1)^2.$$

It is further given that L is a tangent to C at the point P.

Determine the possible coordinates of P.

8.

The straight line with equation

$$y = 2x + k,$$

where k is constant, is a tangent to the curve with equation

$$y = x^2 - 8x + 1.$$

By **using the discriminant** of a suitable quadratic, determine the value of the constant k and hence find the point of contact between the tangent and the curve.

Simultaneous Equations Solutions

1.

a $x + 2 = x^2 - 4$

$x^2 - x - 6 = 0$

$(x + 2)(x - 3) = 0$

$x = -2$ or 3

$\therefore$ $(-2, 0)$ and $(3, 5)$

b $4x + 11 = x^2 + 3x - 1$

$x^2 - x - 12 = 0$

$(x + 3)(x - 4) = 0$

$x = -3$ or 4

$\therefore$ $(-3, -1)$ and $(4, 27)$

c $2x - 1 = 2x^2 + 3x - 7$

$2x^2 + x - 6 = 0$

$(2x - 3)(x + 2) = 0$

$x = -2$ or $\frac{3}{2}$

$\therefore$ $(-2, -5)$ and $(\frac{3}{2}, 2)$

2.

$5 - x = x^2 - 3x + 2$

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x = -1$ or 3

P and Q are the points $(-1, 6)$ and $(3, 2)$

$PQ^2 = (3 + 1)^2 + (2 - 6)^2$

$PQ = \sqrt{32} = 4\sqrt{2}$

3.

$3^{x-1} = (3^2)^{2y}$ $\quad\therefore\ x - 1 = 4y$

$(2^3)^{x-2} = (2^2)^{1+y}$ $\quad\therefore\ 3x - 6 = 2 + 2y$

$6x - 16 = 4y$

$\Rightarrow \quad 6x - 16 = x - 1$

$x = 3$

$\therefore \quad x = 3, y = \frac{1}{2}$

4.

$$\left.\begin{aligned} y &= 1 - x \\ y &= x^2 - 6x + 10 \end{aligned}\right\} \Rightarrow x^2 - 6x + 10 = 1 - x$$

$$\Rightarrow x^2 - 5x + 9 = 0$$

$b^2 - 4ac = (-5)^2 - 4 \times 1 \times 9 = 25 - 36 = -11 < 0$

NO REAL SOLUTIONS

NO INTERSECTIONS BETWEEN THE GRAPHS

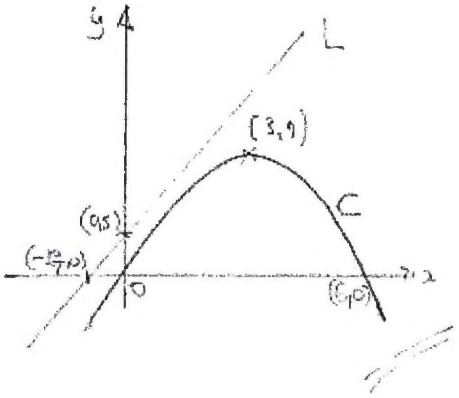
5.

(a) $\left.\begin{array}{l} y = x(6-x) \\ 2y = 7x+10 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 2y = 2x(6-x) \\ 2y = 7x+10 \end{array}\right\} \Rightarrow$

$$2x(6-x) = 7x+10$$
$$12x - 2x^2 = 7x + 10$$
$$0 = 2x^2 - 5x + 10$$
$$b^2 - 4ac = (-5)^2 - 4\times2\times10$$
$$= 25 - 80 = -55 < 0$$

HENCE NO SOLUTIONS TO THE QUADRATIC, SO L & C DO NOT INTERSECT

(b)



- $2y = 7x+10$ $\quad x=0 \quad y=5$
  $\quad y=0 \quad x=-\frac{10}{7}$
- $y = x(6-x)$
  $y=0 \quad x = 0$ or 6
  $x=3$ (LINE OF SYMMETRY)
  $y=9$) IT MAX $(3,9)$

6.

$\left.\begin{array}{l} y = 4x^2 - 7x + 11 \\ y = 5x + k \end{array}\right\}$

$\Rightarrow 4x^2 - 7x + 11 = 5x + k$
$\Rightarrow 4x^2 - 7x - 5x + 11 - k = 0$
$\Rightarrow 4x^2 - 12x + 11 - k = 0$

TWO INTERSECTIONS $b^2 - 4ac > 0$

$\Rightarrow (-12)^2 - 4\times4\times(11-k) > 0$
$\Rightarrow 144 - 16(11-k) > 0$ (DIVIDE THROUGH BY 16)
$\Rightarrow 9 - (11-k) > 0$
$\Rightarrow -2 + k > 0$
$\Rightarrow k > 2$

AS REQUIRED

7.

$C: y = 3(x+1)^2$

$L: y = kx - 9$

$\Rightarrow 3(x+1)^2 = kx - 9$

$\Rightarrow 3x^2 + 6x + 3 = kx - 9$

$\Rightarrow \boxed{3x^2 + (6-k)x + 12 = 0}$ (*)

$\Rightarrow$ TANGENT $\Rightarrow$ REPEATED ROOTS

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (6-k)^2 - 4 \times 3 \times 12 = 0$

$\Rightarrow k^2 - 12k + 36 - 144 = 0$

$\Rightarrow k^2 - 12k - 108 = 0$

$\Rightarrow (k+6)(k-18) = 0$

$k = -6$ or $k = 18$

• If $k = -6$

(*) $\Rightarrow 3x^2 + 12x + 12 = 0$

$\Rightarrow x^2 + 4x + 4 = 0$

$\Rightarrow (x+2)^2 = 0$

$x = -2$

$\downarrow$

$y = 3(x+1)^2$

$y = 3$

$\therefore P(-2, 3)$

• If $k = 18$

(*) $\Rightarrow 3x^2 - 12x + 12 = 0$

$\Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow (x-2)^2 = 0$

$x = 2$

$\downarrow$

$y = 3(x+1)^2$

$y = 27$

$P(2, 27)$

8.

$$\left.\begin{array}{l} y = 2x + k \\ y = x^2 - 8x + 1 \end{array}\right\} \Rightarrow x^2 - 8x + 1 = 2x + k$$

$$\Rightarrow x^2 - 10x + 1 - k = 0$$

$$\Rightarrow \boxed{x^2 - 10x + (1-k) = 0}$$

FOR A TANGENT THIS QUADRATIC MUST PRODUCE REPEATED ROOTS

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow (-10)^2 - 4 \times 1 \times (1-k) = 0$$

$$\Rightarrow 100 - 4(1-k) = 0$$

$$\Rightarrow 100 - 4 + 4k = 0$$

$$\Rightarrow 4k = -96$$

$$\Rightarrow k = -24$$

USING $k = -24$ INTO $x^2 - 10x + (1-k) = 0$

$$x^2 - 10x + 25 = 0$$

$$(x-5)^2 = 0$$

$$\boxed{x = 5}$$

& USING $y = 2x - 24$

$$\boxed{y = -14}$$

$(5, -14)$